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ORDERING POLICY FOR PERISHABLE PRODUCTS WITH LIFE TIME UNDER TRADE CREDIT AND TIME DISCOUNTING

Hari Kishan
Department of Mathematics,
D.N. College
Meerut(U.P.)

Megha Rani
Department of Mathematics
RKGIT
Ghaziabad (U.P.)

Vipin Kumar
Research Scholar
Mewar University
Gangrar, Chittorgarh (Raj.)

Abstract:
In this paper, an inventory model of deteriorating products with life time has been discussed. Demand rate has been taken of quadratic form which starts from zero in the beginning and ends to zero at the completion of the cycle. Deterioration rate has been taken constant. Model has been developed under trade credit and time discounting. Shortages are not considered. Cost minimization technique has been used in the development of the model.

Key-words: Deterioration, life time, trade credit and time discounting.

1 Introduction:

In the classical EOQ model, it is assumed that the retailer must be paid for the items at the time of delivery. However, in real life situation, the supplier may offer the retailer a delay period, which is called the trade credit period, for the payment of purchasing cost to stimulate his products. During the trade credit period, the retailer can sell the products and can earn the interest on the revenue thus obtained. It is beneficial for the retailer to delay the settlement of the replenishment account up to the last moment of the permissible delay allowed by the supplier.

Several researchers discussed the inventory problems under the permissible delay in payment condition. **Goyal** (1985) discussed a single item inventory model under permissible delay in payment. **Chung** (1998) used an alternative approach to obtain the economic order quantity under permissible delay in payment. **Agrawal & Jaggi** (1995) discussed the inventory model with an exponential deterioration rate under permissible delay in payment. **Chang et. al.** (2002) extended this work for variable deterioration rate. **Liao, et al.** (2000) discussed the topic with inflation. **Jamal, et. al**. (1997) and **Chang & Dye** (2001) extended this work with shortages. **Chen & Chung** (1999) discussed light buyer's economic order model under trade credit. **Huang & Shinn** (1997) studied an inventory system for retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payment. **Jamal, et. al.** (2000) and **Sarkar, et. al.** (2000) obtained the optimal time of payment under permissible delay in payment. **Teng** (2002) considered the selling price not equal to the purchasing price to modify the model under permissible delay in payment. **Shinn & Huang** (2003) obtained the optimal price and order size simultaneously under the condition of order-size dependent delay in payments. They assumed the length of the credit period as a function of retailer's order size and the demand rate to be the function of selling price. **Chung & Huang** (2003) extended this work within the EPQ framework and obtained the retailer's optimal ordering policy. **Huang** (2003) extended this work under two level of trade credit. **Huang** (2005) modify the **Goyal's** model under the following assumptions:

(i) The unit selling price and the unit purchasing price are not necessarily equal.
(ii) The supplier offers the retailer partial trade credit, i.e. the retailer has to make a partial payment to the supplier in the beginning and has to pay the remaining balance at the end of the permissible delay period.

He adopted the cost minimization technique to investigate the optimal retailer's inventory policy. **Megha Rani, Hari Kishan and Shiv Raj Singh** (2011) discussed the inventory model of deteriorating products under supplier's partial trade credit policy. **Hari Kishan, Megha Rani and Sarvendar Saroha** (2010) discussed an inventory model of decaying products with life time and variational demand. **Hari Kishan, Megha Rani and Deep Shikha (2012)** discussed the inventory model of deteriorating products with life time under declining demand and permissible delay in payment. **Hari Kishan, Megha Rani and Deep Shikha** (2013) developed the inventory model with variable demand, shortages and deterioration.
While determining the optimal ordering policy, the effect of inflation and time value of money cannot be ignored. **Buzacott** (1975) developed an EOQ model with inflation subject to different types of pricing policies. **Hou** (2006) developed an inventory model with deterioration, inflation, time value of money and stock dependent consumption rate. **Hou and Lin** (2006) extended the work of **Hou** (2006) by considering the selling rate as a function of stock level and selling price over the finite time horizon. **Chandra and Bahner** (1985), **Ray and Chaudhary** (1997), **Chung and Lin** (2001), **Wee and Law** (2001) and **Balkhi** (2004), also worked in this direction. **Hou and Lin** (2008) developed an ordering policy for deteriorating items under trade credit and time discounting.
In most of the inventory models it is assumed that the deterioration of items starts in the very beginning of the inventory. In real life problems the deterioration of items starts after some time which is known as life time. In this paper, an inventory model has been developed for deteriorating items with life time under the assumptions of life time, trade credit and time discounting. The work of **Hou and Lin** (2008) has been extended with the case of life time and variable demand rate of quadratic form.

2 Assumptions and Notations:

Assumptions:

The following assumptions are considered in this paper:
(i) The demand rate is quadratic function of time which is given by $-at(T-t)$.
(ii) Time horizon is finite given by H.
(iii) Shortages are not allowed.
(iv) The deteriorating rate is deterministic and constant. Deterioration starts after time μ which is the life time.
(vi) During the time the account is not settled, the generated sales revenue is deposited in an interest bearing account. When $T \geq M$, the account is settled at $T=M$ and we start paying for the interest charges on items in stock. When $T \leq M$, the account is settled at $T=M$ and we need not to pay any interest charge.

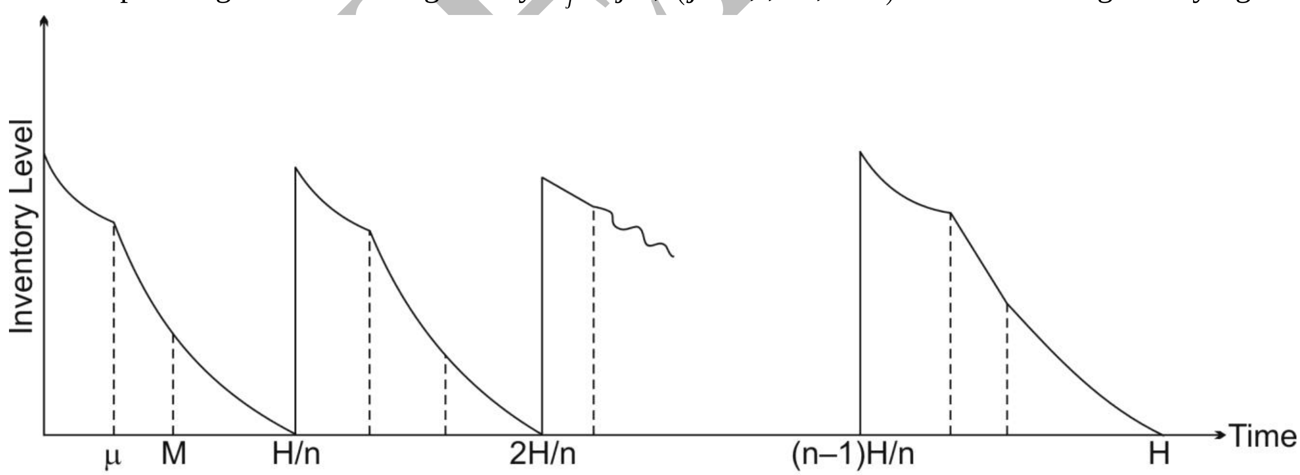
Notations:

The following notations have been used in this chapter:
(i) The demand rate= $-at(T-t)$ where a positive constant.
(ii) A = the ordering cost per order.
(iii) c= the unit purchasing price.
(iv) h=unit holding cost per unit time excluding interest charges.
(v) M= the trade credit period.
(vi) H= length of planning horizon.

(vii) T=the replenishment cycle time in years.
(viii) n= number of replenishment during the planning horizon.
(ix) θ =the deterioration rate of the on hand inventory.
(x) μ = the life time.
(xi) r= discount rate representing the time value of money.
(xii) f= inflation rate
(xiii) R= the net discount rate of inflation.
(xiv) I_e = interest earned per Re per year.
(xv) I_c = interest charged per Re per year.
(xvi) $q(t)$= stock level at any time t.
(xvii)Q= maximum stock level.
(xviii)s=selling price per unit
(xix) $TVC(T)$= the annual total relevant cost, which is a function of T.
(xx) T^* = the optimal cycle time of $TVC(T)$.
(xxi) Q^* = the optimal order quantity.

3 Mathematical Model:

The total time horizon H has been divided in n equal parts of length T so that $T=\frac{H}{n}$. Therefore the reorder times over the planning horizon H are given by $T_j = jT$, $(j = 0,1,2...,n-1)$. This model is given by fig. 1.



(Fig. 1)

Let $q(t)$ be the inventory level during the first replenishment cycle. This inventory level is depleted due to demand during life time and due to demand and deterioration after life time. The governing differential equations of the stock status during the period $0 \le t \le T$ are given by

$$\frac{dq}{dt} = -at(T-t), \qquad 0 \le t \le \mu \qquad \ldots(1)$$

$$\frac{dq}{dt} + \theta q = -at(T-t). \qquad \mu \le t \le T \qquad \ldots(2)$$

The boundary conditions are

$$q(0)=Q, \quad \ldots(3)$$

$$q(T)=0. \quad \ldots(4)$$

4 Analysis:

Solving equation (1) and using boundary condition (3), we get

$$q=Q-a\left[\frac{Tt^2}{2}-\frac{t^3}{3}\right]. \quad \ldots(5)$$

Solving equation (2) and using boundary condition (4), we get

$$q=-a\left[t(T-t)\frac{1}{\theta}-(T-2t)\frac{1}{\theta^2}-\frac{2}{\theta^3}\right]+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]e^{\theta(T-t)}. \quad \ldots(6)$$

The continuity of $q(t)$ at $t=\mu$ gives

$$q(\mu)=Q-a\left[\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right]$$

$$=-a\left[\mu(T-\mu)\frac{1}{\theta}-(T-2\mu)\frac{1}{\theta^2}-\frac{2}{\theta^3}\right]+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]e^{\theta(T-\mu)}.$$

This provides

$$q(0)=Q=a\left[\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right]-a\left[\mu(T-\mu)\frac{1}{\theta}-(T-2\mu)\frac{1}{\theta^2}-\frac{2}{\theta^3}\right]$$

$$+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]e^{\theta(T-\mu)}. \quad \ldots(7)$$

The present value of the total replenishment costs is given by

$$C_R=A\sum_{j=0}^{n-1}e^{-jRT}$$

$$=A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \quad \ldots(8)$$

The present value of the total purchasing costs is given by

$$C_p=c\sum_{j=0}^{n-1}q(0)e^{-jRT}$$

$$=c\left[a\left[\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right]-a\left[\mu(T-\mu)\frac{1}{\theta}-(T-2\mu)\frac{1}{\theta^2}-\frac{2}{\theta^3}\right]\right.$$

$$\left.+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]e^{\theta(T-\mu)}\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}. \quad \ldots(9)$$

The present value of the holding cost during the first replenishment cycle is given by

$$h_1=h\left[\int_0^{\mu}q(t)e^{-Rt}dt+\int_{\mu}^{T}q(t)e^{-Rt}dt\right]$$

$$= h\left[\int_0^{\mu}\left\{Q - a\left[\frac{Tt^2}{2} - \frac{t^3}{3}\right]\right\}e^{-Rt}dt\right.$$

$$\left.+\int_{\mu}^{T} - a\left[t(T-t)\frac{1}{\theta} - (T-2t)\frac{1}{\theta^2} - \frac{2}{\theta^3}\right] + a\left[\frac{T}{\theta^2} - \frac{2}{\theta^3}\right]e^{\theta(T-t)}e^{-Rt}\,dt\right]$$

$$= h\left[\frac{Q}{R}\left(1-e^{-R\mu}\right) + a\left[\left(\frac{T}{2} - \frac{\mu}{3}\right)\frac{\mu^2 e^{-R\mu}}{R} + (T-\mu)\frac{\mu e^{-R\mu}}{R^2} + (T-2\mu)\frac{e^{-R\mu}}{R^3} + \frac{2e^{-R\mu}}{R^4}\right]\right.$$

$$-a\left[\frac{Te^{-RT}}{\theta R^2} + \frac{2e^{-RT}}{\theta R^3} + (T-\mu)\frac{\mu e^{-RT}}{\theta R} + (T-2\mu)\frac{e^{-RT}}{\theta R^2} - \frac{2e^{-RT}}{\theta R^3}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^2 R} + \frac{2e^{-RT}}{\theta^2 R^2} + (T-2\mu)\frac{e^{-R\mu}}{\theta^2 R} - \frac{2e^{-R\mu}}{\theta^2 R^2} - \frac{2e^{-RT}}{\theta^3 R} + \frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$\left.+a\left[\frac{T}{\theta^2} - \frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu} - e^{-RT}}{R+\theta}\right]\right]. \qquad \ldots(10)$$

The present value of the total holding cost is given by

$$C_h = \sum_{j=0}^{n-1} h_1 e^{-jRT}$$

$$= h\left[\frac{Q}{R}\left(1-e^{-R\mu}\right) + a\left[\left(\frac{T}{2} - \frac{\mu}{3}\right)\frac{\mu^2 e^{-R\mu}}{R} + (T-\mu)\frac{\mu e^{-R\mu}}{R^2} + (T-2\mu)\frac{e^{-R\mu}}{R^3} + \frac{2e^{-R\mu}}{R^4}\right]\right.$$

$$-a\left[\frac{Te^{-RT}}{\theta R^2} + \frac{2e^{-RT}}{\theta R^3} + (T-\mu)\frac{\mu e^{-RT}}{\theta R} + (T-2\mu)\frac{e^{-RT}}{\theta R^2} - \frac{2e^{-RT}}{\theta R^3}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^2 R} + \frac{2e^{-RT}}{\theta^2 R^2} + (T-2\mu)\frac{e^{-R\mu}}{\theta^2 R} - \frac{2e^{-R\mu}}{\theta^2 R^2} - \frac{2e^{-RT}}{\theta^3 R} + \frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$\left.+a\left[\frac{T}{\theta^2} - \frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu} - e^{-RT}}{R+\theta}\right]\right] \times \left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right). \qquad \ldots(11)$$

Now we consider the following two cases:

Case 1: $M \le T = \dfrac{H}{n}$.

Sub case I: $0 \prec M \le \mu$.

In this case, the interest payable is given by

$$I_{p1}^{1} = cI_c\left[\int_M^{\mu} q(t)e^{-Rt}dt + \int_{\mu}^{T} q(t)e^{-Rt}dt\right]$$

$$= cI_c\left[\frac{Q}{R}\left(e^{-RM} - e^{-R\mu}\right) + a\left\{\left(\frac{T\mu^2}{2} - \frac{\mu^3}{3}\right)\frac{e^{-R\mu}}{R} - \left(\frac{TM^2}{2} - \frac{M^3}{3}\right)\frac{e^{-RM}}{R}\right.\right.$$

$$\left.+\left(T\mu - \mu^2\right)\frac{e^{-R\mu}}{R^2} - \left(TM - M^2\right)\frac{e^{-RM}}{R^2} + (T-2\mu)\frac{e^{-R\mu}}{R^3} - (T-2M)\frac{e^{-RM}}{R^3} + \frac{2e^{-R\mu}}{R^4} - \frac{2e^{-RM}}{R^4}\right\}$$

$$-a\left[\frac{Te^{-RT}}{\theta R^2}+\frac{2e^{-RT}}{\theta R^3}+(T-\mu)\frac{\mu e^{-RT}}{\theta R}+(T-2\mu)\frac{e^{-RT}}{\theta R^2}-\frac{2e^{-RT}}{\theta R^3}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^2 R}+\frac{2e^{-RT}}{\theta^2 R^2}+(T-2\mu)\frac{e^{-R\mu}}{\theta^2 R}-\frac{2e^{-R\mu}}{\theta^2 R^2}-\frac{2e^{-RT}}{\theta^3 R}+\frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu}-e^{-RT}}{R+\theta}\right]\Bigg]. \qquad \ldots(12)$$

The present value of the total interest payable over the time horizon H is given by

$$I_{p1}^{H}=\sum_{j=0}^{n-1} I_{p1}^{1} e^{-jRT}$$

$$=cI_c\Bigg[\frac{Q}{R}\left(e^{-RM}-e^{-R\mu}\right)+a\Bigg\{\left(\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right)\frac{e^{-R\mu}}{R}-\left(\frac{TM^2}{2}-\frac{M^3}{3}\right)\frac{e^{-RM}}{R}$$

$$+\left(T\mu-\mu^2\right)\frac{e^{-R\mu}}{R^2}-\left(TM-M^2\right)\frac{e^{-RM}}{R^2}+(T-2\mu)\frac{e^{-R\mu}}{R^3}-(T-2M)\frac{e^{-RM}}{R^3}$$

$$+\frac{2e^{-R\mu}}{R^4}-\frac{2e^{-RM}}{R^4}\Bigg\} \qquad -a\left[\frac{Te^{-RT}}{\theta R^2}+\frac{2e^{-RT}}{\theta R^3}+(T-\mu)\frac{\mu e^{-RT}}{\theta R}+(T-2\mu)\frac{e^{-RT}}{\theta R^2}-\frac{2e^{-RT}}{\theta R^3}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^2 R}+\frac{2e^{-RT}}{\theta^2 R^2}+(T-2\mu)\frac{e^{-R\mu}}{\theta^2 R}-\frac{2e^{-R\mu}}{\theta^2 R^2}-\frac{2e^{-RT}}{\theta^3 R}+\frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu}-e^{-RT}}{R+\theta}\right]\Bigg]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right). \qquad \ldots(13)$$

The present value of the total interest earned during the first replenishment cycle is given by

$$I_{e1}^{1}=sI_e\int_0^T at(T-t)te^{-Rt}dt$$

$$=sI_e\left[\frac{T^2e^{-RT}}{R^2}-\frac{2Te^{-RT}}{R^3}+\frac{6}{R^4}\left(1-e^{-RT}\right)\right]. \qquad \ldots(14)$$

Hence the present value of the total interest earned over the time horizon H is given by

$$I_{e1}^{H}=\sum_{j=0}^{n-1} I_{e1}^{1} e^{-jRT}$$

$$=sI_e\left[\frac{T^2e^{-RT}}{R^2}-\frac{2Te^{-RT}}{R^3}+\frac{6}{R^4}\left(1-e^{-RT}\right)\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right).$$

Sub case II: $\mu \le M$ **.**

In this case, the interest payable is given by

$$I_{p1}^{2} = cI_{c}\left[\int_{M}^{T} q(t)e^{-Rt}dt\right]$$

$$= cI_{C}\left[-a\left[\frac{Te^{-RT}}{\theta R^{2}}+\frac{2e^{-RT}}{\theta R^{3}}+(T-M)\frac{Me^{-RT}}{\theta R}+(T-2M)\frac{e^{-RT}}{\theta R^{2}}-\frac{2e^{-RT}}{\theta R^{3}}\right]\right.$$

$$+a\left[\frac{Te^{-RT}}{\theta^{2}R}+\frac{2e^{-RT}}{\theta^{2}R^{2}}+(T-2M)\frac{e^{-RM}}{\theta^{2}R}-\frac{2e^{-RM}}{\theta^{2}R^{2}}-\frac{2e^{-RT}}{\theta^{3}R}+\frac{2e^{-RM}}{\theta^{3}R}\right]$$

$$\left.+a\left[\frac{T}{\theta^{2}}-\frac{2}{\theta^{3}}\right]\left[\frac{e^{\theta(T-M)-RM}-e^{-RT}}{R+\theta}\right]\right]. \qquad \ldots(15)$$

The present value of the total interest payable over the time horizon H is given by

$$I_{p1}^{H} = \sum_{j=0}^{n-1} I_{p1}^{2} e^{-jRT}$$

$$= cI_{C}\left[-a\left[\frac{Te^{-RT}}{\theta R^{2}}+\frac{2e^{-RT}}{\theta R^{3}}+(T-M)\frac{Me^{-RT}}{\theta R}+(T-2M)\frac{e^{-RT}}{\theta R^{2}}-\frac{2e^{-RT}}{\theta R^{3}}\right]\right.$$

$$+a\left[\frac{Te^{-RT}}{\theta^{2}R}+\frac{2e^{-RT}}{\theta^{2}R^{2}}+(T-2M)\frac{e^{-RM}}{\theta^{2}R}-\frac{2e^{-RM}}{\theta^{2}R^{2}}-\frac{2e^{-RT}}{\theta^{3}R}+\frac{2e^{-RM}}{\theta^{3}R}\right]$$

$$\left.+a\left[\frac{T}{\theta^{2}}-\frac{2}{\theta^{3}}\right]\left[\frac{e^{\theta(T-M)-RM}-e^{-RT}}{R+\theta}\right]\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right). \qquad \ldots(16)$$

Therefore the total present value of the costs over the time horizon H is given by

$$TVC_{1}(n) = C_{R}+C_{p}+C_{h}+I_{p1}^{H}-I_{e1}^{H}$$

$$= A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}+c\left[a\left[\frac{T\mu^{2}}{2}-\frac{\mu^{3}}{3}\right]-a\left[\mu(T-\mu)\frac{1}{\theta}-(T-2\mu)\frac{1}{\theta^{2}}-\frac{2}{\theta^{3}}\right]\right.$$

$$\left.+a\left[\frac{T}{\theta^{2}}-\frac{2}{\theta^{3}}\right]e^{\theta(T-\mu)}\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}$$

$$+h\left[\frac{Q}{R}\left(1-e^{-R\mu}\right)+a\left[\left(\frac{T}{2}-\frac{\mu}{3}\right)\frac{\mu^{2}e^{-R\mu}}{R}+(T-\mu)\frac{\mu e^{-R\mu}}{R^{2}}+(T-2\mu)\frac{e^{-R\mu}}{R^{3}}+\frac{2e^{-R\mu}}{R^{4}}\right]\right.$$

$$-a\left[\frac{Te^{-RT}}{\theta R^{2}}+\frac{2e^{-RT}}{\theta R^{3}}+(T-\mu)\frac{\mu e^{-RT}}{\theta R}+(T-2\mu)\frac{e^{-RT}}{\theta R^{2}}-\frac{2e^{-RT}}{\theta R^{3}}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^{2}R}+\frac{2e^{-RT}}{\theta^{2}R^{2}}+(T-2\mu)\frac{e^{-R\mu}}{\theta^{2}R}-\frac{2e^{-R\mu}}{\theta^{2}R^{2}}-\frac{2e^{-RT}}{\theta^{3}R}+\frac{2e^{-R\mu}}{\theta^{3}R}\right]$$

$$\left.+a\left[\frac{T}{\theta^{2}}-\frac{2}{\theta^{3}}\right]\left[\frac{e^{\theta(T-\mu)-R\mu}-e^{-RT}}{R+\theta}\right]\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right)$$

$$+cI_c\left[\frac{Q}{R}\left(e^{-RM}-e^{-R\mu}\right)+a\left\{\left(\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right)\frac{e^{-R\mu}}{R}-\left(\frac{TM^2}{2}-\frac{M^3}{3}\right)\frac{e^{-RM}}{R}\right.\right.$$

$$+\left(T\mu-\mu^2\right)\frac{e^{-R\mu}}{R^2}-\left(TM-M^2\right)\frac{e^{-RM}}{R^2}+\left(T-2\mu\right)\frac{e^{-R\mu}}{R^3}-\left(T-2M\right)\frac{e^{-RM}}{R^3}$$

$$\left.+\frac{2e^{-R\mu}}{R^4}-\frac{2e^{-RM}}{R^4}\right\}\quad -a\left[\frac{Te^{-RT}}{\theta R^2}+\frac{2e^{-RT}}{\theta R^3}+\left(T-\mu\right)\frac{\mu e^{-RT}}{\theta R}+\left(T-2\mu\right)\frac{e^{-RT}}{\theta R^2}-\frac{2e^{-RT}}{\theta R^3}\right]$$

$$+a\left[\frac{Te^{-RT}}{\theta^2 R}+\frac{2e^{-RT}}{\theta^2 R^2}+\left(T-2\mu\right)\frac{e^{-R\mu}}{\theta^2 R}-\frac{2e^{-R\mu}}{\theta^2 R^2}-\frac{2e^{-RT}}{\theta^3 R}+\frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$\left.+a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu}-e^{-RT}}{R+\theta}\right]\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right)$$

$$-sI_e\left[\frac{T^2e^{-RT}}{R^2}-\frac{2Te^{-RT}}{R^3}+\frac{6}{R^4}\left(1-e^{-RT}\right)\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right). \qquad \ldots(17)$$

Case 2: $M > T = \frac{H}{n}$**:**

In this case, the interest charged by the supplier will be zero because the supplier can be paid at M which is greater than T. The interest earned in the first cycle is the interest earned during the period $(0,T)$ and the interest earned from the cash invested during the period (T,M). This is given by

$$I_{e2}^1 = sI_e\left[\int_0^T at(T-t)te^{-Rt}dt+(M-T)e^{-RT}\int_T^M at(T-t)tdt\right]$$

$$=asI_e\left[\left(\frac{T^2}{R^2}+\frac{4T}{R^3}-\frac{6}{R^4}\right)e^{-RT}+\left(\frac{2T}{R^3}+\frac{6}{R^4}\right)\right.$$

$$\left.+(M-T)e^{-RT}\left(\frac{TM^3}{3}-\frac{M^4}{4}-\frac{T^4}{12}\right)\right]. \qquad \ldots(18)$$

Hence the present value of the total interest earned over the time horizon H is given by

$$I_{e2}^H=\sum_{j=0}^{n-1}I_{e2}^1 e^{-jRT}$$

$$=asI_e\left[\left(\frac{T^2}{R^2}+\frac{4T}{R^3}+\frac{6}{R^4}\right)e^{-RT}+\left(\frac{2T}{R^3}-\frac{6}{R^4}\right)\right.$$

$$\left.+(M-T)e^{-RT}\left(\frac{TM^3}{3}-\frac{M^4}{4}-\frac{T^4}{12}\right)\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right). \qquad \ldots(19)$$

As the replenishment cost, purchasing cost and holding cost over the time horizon H are the same as in case I, the total present value of the costs is given by

$$TVC_2(n)=C_R+C_p+C_h-I_{e2}^H$$

$$= A\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)} + c\left[a\left[\frac{T\mu^2}{2}-\frac{\mu^3}{3}\right] - a\left[\mu(T-\mu)\frac{1}{\theta} - (T-2\mu)\frac{1}{\theta^2} - \frac{2}{\theta^3}\right]\right.$$

$$\left. + a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]e^{\theta(T-\mu)}\right]\frac{\left(1-e^{-RH}\right)}{\left(1-e^{-\frac{RH}{n}}\right)}$$

$$+ h\left[\frac{Q}{R}\left(1-e^{-R\mu}\right) + a\left[\left(\frac{T}{2}-\frac{\mu}{3}\right)\frac{\mu^2 e^{-R\mu}}{R} + (T-\mu)\frac{\mu e^{-R\mu}}{R^2} + (T-2\mu)\frac{e^{-R\mu}}{R^3} + \frac{2e^{-R\mu}}{R^4}\right]\right.$$

$$- a\left[\frac{Te^{-RT}}{\theta R^2} + \frac{2e^{-RT}}{\theta R^3} + (T-\mu)\frac{\mu e^{-RT}}{\theta R} + (T-2\mu)\frac{e^{-RT}}{\theta R^2} - \frac{2e^{-RT}}{\theta R^3}\right]$$

$$+ a\left[\frac{Te^{-RT}}{\theta^2 R} + \frac{2e^{-RT}}{\theta^2 R^2} + (T-2\mu)\frac{e^{-R\mu}}{\theta^2 R} - \frac{2e^{-R\mu}}{\theta^2 R^2} - \frac{2e^{-RT}}{\theta^3 R} + \frac{2e^{-R\mu}}{\theta^3 R}\right]$$

$$\left. + a\left[\frac{T}{\theta^2}-\frac{2}{\theta^3}\right]\left[\frac{e^{\theta(T-\mu)-R\mu} - e^{-RT}}{R+\theta}\right]\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right)$$

$$- asI_e\left[\left(\frac{T^2}{R^2} + \frac{4T}{R^3} + \frac{6}{R^4}\right)e^{-RT} + \left(\frac{2T}{R^3} - \frac{6}{R^4}\right)\right.$$

$$\left. + (M-T)e^{-RT}\left(\frac{TM^3}{3} - \frac{M^4}{4} - \frac{T^4}{12}\right)\right]\left(\frac{1-e^{-RH}}{1-e^{-RH/n}}\right) \qquad \ldots(20)$$

At $M=T=\frac{H}{n}$, we have $TVC_1(n)=TVC_2(n)$. Thus we have

$$TVC(n) = \begin{cases} TVC_1(n), T = \frac{H}{n} \geq M \\ TVC_2(n), T = \frac{H}{n} \leq M \end{cases}$$

5 Algorithm: The following algorithm is used to derive the optimal values of n, T, Q and $TVC(n)$:

Step1. Start by choosing a discrete value of n equal or greater than 1.

Step2. If $T=\frac{H}{n} \geq M$ for different integral values of n then $TVC_1(n)$ is derived from expression (17). If $T=\frac{H}{n} \leq M$ for different integral values of n then $TVC_2(n)$ is derived from expression (20).

Step3. Step 1 and 2 are repeated for all possible values of n with $T=\frac{H}{n} \geq M$ until the minimum value of $TVC_1(n)$ is found from expression (17). Let $n=n_1^*$ be such value of n. For all possible values of n with $T=\frac{H}{n} \leq M$ until the minimum value of $TVC_2(n)$ is found from expression (20). Let $n=n_2^*$ be such value of n. The values $n_1^*, n_2^*, TVC_1(n^*)$ and $TVC_2(n^*)$ constitute the optimal solution.

Step4. The optimal number of replenishment n^* is selected such that

$$TVC(n^*) = \begin{cases} TVC_1(n), T = \dfrac{H}{n^*} \geq M \\ TVC_2(n), T = \dfrac{H}{n^*} \leq M \end{cases}$$

The optimal value of ordered quantity Q^* is derived by substituting n^* in the expression (7) and optimal cycle time T^* is given by $T = \dfrac{H}{n^*}$.

6 Conclusions:

In this paper, an inventory model has been developed for deteriorating items with life time under the assumptions of trade credit and time discounting. Time horizon has been considered finite. The demand rate has been taken quadratic function of time starting from zero and ending with zero. The deterioration rate has been taken constant. The time horizon has been divided into *n* equal sub-intervals. This work can further be extended for other forms of demand rate, for variable deterioration rate and for multi items.

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